PMM U.S.S.R., Vol.45,pp.555-556 0021-8928/82/4 0555 $7.50/0

UDC 531.38

PERIODIC SOLUTIONS OF THE PROBLEM OF MOTION OF A HEAVY RIGID BODY ABOUT A FIXED POINT IN THE KOVALEVSKAIA CASE*

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Method of point transformations is used to study the periodic solutions of the problem of motion of a heavy rigid body about a fixed point in the Kovalevskaia case, using the nonholonomic Kolosov variables. The method of point transformations used to obtain periodic solutions originates in the works of Poincaré. According to the method, instead of considering a complete trajectory in the phase space, one considers its consecutive intersections with a specified surface, in particular with a plane. Thus the trajectories appear and are studied on this plane of surface, with considerable geometrical simplification. The method was first used in the rigid body dynamics /1/ where it was applied to the equations of motion in the isothermic coordinates defined on the inertia ellipsoid /2/.

We transform the Euler– Poisson system of equations to the equation of plane motion of a fictitious material point acted upon by a potential force, by introducing nonholonomic coordinates x, y and a new, independent variable τ /3/

$$\frac{d^2x}{d\tau^2} = \frac{\partial U}{\partial x}, \quad \frac{d^2y}{d\tau^2} = \frac{\partial U}{\partial y}, \quad \left(d\tau = \frac{r^2(p^2+q^2)+2rp\gamma_3+\gamma_3^2}{2q^2}\,dt\right) \tag{1}$$

Here $p, q, r, \gamma_1, \gamma_2, \gamma_3$ denote the angular velocity components in the associated coordinate system and the direction cosines of the vertical, respectively. The force function of the problem is given by the formula

$$U = -\frac{x^2+y^2-kx+1}{\sqrt{x^2+y^2}} \tag{2}$$

Equations of motion (2) admit the kinetic energy integral

$$v^2 = 2(U+C) \tag{3}$$

where C is an arbitrary constant and k is the constant of the Kovalevskaia integral.

The solution of the system (1) represents a trajectory in the phase space x, x', y, y'. The magnitude of the energy constant C must be preserved along the trajectory, therefore if the value of C is given and three coordinates of a point on the phase trajectory are known for some instant of time, then the fourth phase coordinate can be found from the relations (3). Thus we can study, for a fixed value of C, the trajectories of the representative (fictitious) point in the three-dimensional subspace x, x', y of the phase space.

Let us inspect, in the three-dimensional space introduced above, the intersections of the phase trajectory with the xx'-plane in the positive direction, i.e. the points of a trajectory satisfying the conditions $y=0, y'>0$. In the xy-plane these points correspond to the intersection of the trajectory of the fictitious point $x=x(\tau), y=y(\tau)$ with the abscissa axis also in the positive direction. Instead of studying the trajectory of the fictitious point itself, we shall investigate the set of points generated in this manner in the xx'-plane. We shall study the simple periodic trajectories, i.e. the trajectories $x=x(\tau), y=y(\tau)$, which close after the first rotation, and intersect the abscissa axis only twice. To simple periodic trajectory shall correspond one point on the xx'- plane. Moreover, we shall limit ourselves to the trajectories in the xx'-plane symmetrical with respect to the Ox-axis and intersecting this axis only at the right angle. The set of simple periodic trajectories possessing quantitatively identical geometrical properties determined by the initial

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*Prikl.Matem.Mekhan,45,No.4,750-752,1981

conditions and the parameters of the problem, can be brought together in a separate class. Every trajectory is conveniently determined in a unique manner by a point in the xC-plane and this opens a possibility of the energetic classification. The points are grouped into lines which are called the characteristic curves of the classes /4,5/. Every point of the characteristic curve has a single corresponding trajectory in the xC-plane.

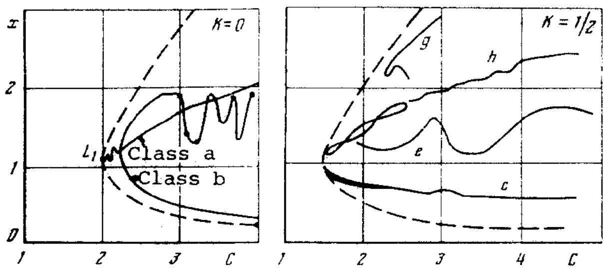

Fig.1

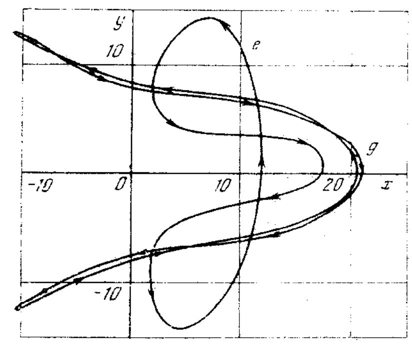

Fig.2

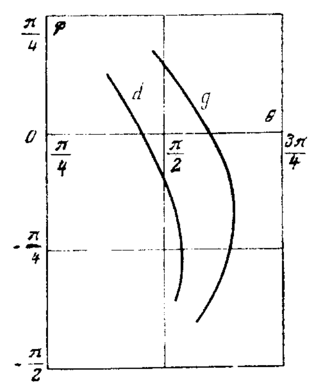

Fig.3

Fig.1 depicts the characteristic curves of the classes of simple periodic trajectories for $C > 0$ in the xC-plane for the Kovalevskaia problem, obtained by numerical integration of the Kolosov equations. We shall not show the complete pattern and will limit ourselves to the case $k = 0$ (classes a and b) and $k = 1/2$ (classes c, e, g and h). We omit for simplicity the perodicity classes for $C < 0$. The extremal dashed lines indicate the Hill zero velocity curves. The point L_1 corresponds to the stationary solutions and is analogous to the libration point in the circular three-body problem. Fig.2 depicts the trajectories of the classes c and g for $k = 1/2$.

Using the initial values of x, x', y and y', the first integrals and the formulas of transformation to the Kolosov variables, we can obtain, for every trajectory, the corresponding initial value of the angle φ of natural rotation and angle ϑ of nutation. Fig.3 shows the curves illustrating the interdependence of the initial values of φ and ϑ for the periodicity classes d and g at $k = 1/2$.

The author thanks V.G. Demin for the guidance and supervision given.

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Translated by L.K.